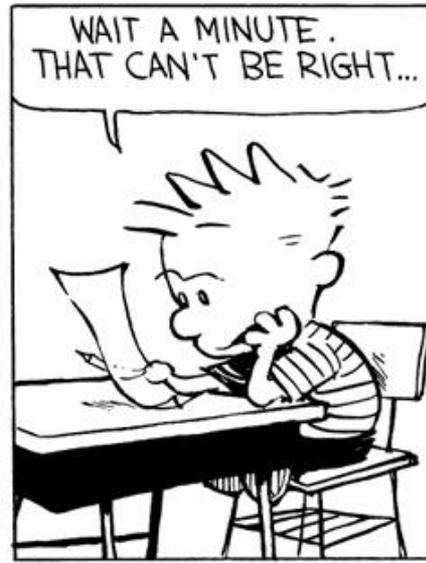
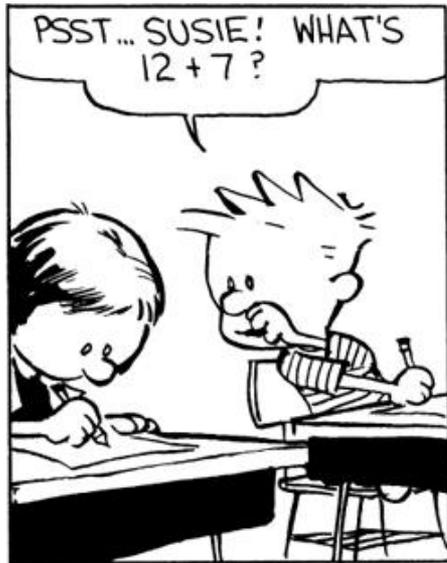


MAT135 Topics in Contemporary Mathematics

Tony Baker

When you think of Math What do you think of?





Quantitative Literacy

Copyright © 2012 by W.H. Freeman and Company

Quantitative Literacy: Thinking Between the Lines

Crauder, Noell, Evans, Johnson

Chapter 1: Critical Thinking

Tony Baker
February 4th, 2013

Chapter 1 Critical Thinking

Lesson Plan

- ▶ Public policy and Simpson's paradox: Is "average" always average?
- ▶ Logic and informal fallacies: Does that argument hold water?
- ▶ Formal logic and truth tables: Do computers think?

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

Learning Objectives:

- ▶ View with a critical eye conclusions based on averages
 - ▶ Understand Simpson's paradox and test scores
 - ▶ Understand the Berkeley gender discrimination case

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

- ▶ **Example:** Suppose a certain high school gave a math proficiency exam to its students and that the percentage who passed was below the statewide average. After examining the figures further, the school decided to report its test data by separating them into students from low-income families and students from higher-income families.

	Local school		Statewide	
	Students tested	Passed	Students tested	Passed
Low income	400	260	200,000	128,000
High Income	700	532	1,100,000	825,000
Total	1100	792	1,300,000	953,000

Show that the local school outperformed statewide students in both the low-income and high-income categories but had a lower passing percentage overall than the statewide rate.

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

- **Solution:** The local school's pass percentage for low-income families:

$$\begin{aligned}\text{Local low - income average} &= \frac{\text{Number passes}}{\text{Numer tests taken}} \times 100\% \\ &= \frac{2260}{200} \times 100\% = 65.0\%.\end{aligned}$$

	Local school % pass	Statewide % pass
Low income	260/400 or 65.0%	128,000/200,000 or 64.0%
High income	532/700 or 76.0%	825,000/1,100,000 or 75.0%
Total	792/1100 or 72.0%	953,000/1,300,000 or 73.3%

Overall the pass rate of the local school is *lower* than the statewide rate by more than one percentage point.

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

▶ **Simpson's paradox**

Simpson's paradox occurs:

1. when combining, or aggregating, data masks underlying patterns.
2. when a factor distorts the overall picture, but distortion goes away when underlying data are examined.

- ▶ **Example:** Separating test scores by the economic level of the students may show that at a local school students at each economic level perform better than the statewide average for students at the same level. But, if school has more students at lower economic levels, its test scores overall may be lower than the state average. Such results can be **counterintuitive**. Without careful consideration, one can be led to an incorrect conclusion.

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

▶ **The Berkeley gender discrimination case:** Data from a 1973 study showed persuasive evidence that the University of California at Berkeley was practicing gender discrimination in graduate school admissions.

	Applicants	Accepted	% accepted
Male	8442	3714	44.0%
Female	4321	1512	35.0%



Students on the University of California at Berkeley campus.

Graduate school admissions are based on departmental selections. Let's study how the male and female applicants might have been divided among two departments, Math and English.

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

	Males			Females		
	Applicants	Accepted	% accepted	Applicants	Accepted	% accepted
Math	2000	500	25.0%	3000	780	26.0%
English	6442	3214	49.9%	1321	732	55.4%
Total	8442	3714	44.0%	4321	1512	35.0%

The two departments actually accepted a larger percentage of female applicants than male applicants.

Most women applied to the Math Department, where it is more difficult to be accepted, but most men applied to English, where it is less difficult.

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

- ▶ **Example:** The following table shows the hitting records of two major league baseball players, Derek Jeter and David Justice, in 1995 and 1996:

	1995		1996	
	At-bats	Hits	At-bats	Hits
Jeter	48	12	582	183
Justice	411	104	140	45

1. Which batter had the higher average in 1995?
2. Which had the higher average in 1996?
3. Which had the higher average over the two-year period?

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

- **Solution:** 1. Jeter's average in 1995 = $\frac{\text{Hits}}{\text{At-bats}} = \frac{12}{48} = 0.250$

We calculate average for Jeter and Justice over each of the two years.

	1995	1996
Average for Jeter	12/48 or 0.250	183/582 or 0.314
Average for Justice	104/411 or 0.253	45/140 or 0.321

2. Jeter's average over two – year peirod = $\frac{\text{Hits}}{\text{At-bats}} = \frac{12+183}{48+582} = \frac{195}{630} = 0.310$

3. Justice gives his average over the two-year period as $\frac{149}{551} = 0.270$. Justice had the higher batting average in each of the two years, but Jeter had the higher batting average over the two-year period.

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

Quick Review Calculating Percentages

To calculate P% of a quantity, we multiply the quantity by $\frac{P}{100}$:

$$P\% \text{ of quantity} = \frac{P}{100} \times \text{Quantity}$$

To find what percentage of a whole a part:

$$\text{Percentage} = \frac{\text{Part}}{\text{Whole}} \times 100\%$$

- ▶ **Example:** Find 45% of 500.

$$45\% \text{ of } 500 = \frac{45}{100} \times 500 = 225.$$

- ▶ **Example:** Find what percentage of 140 is 35.

$$\text{Percentage} = \frac{\text{Part}}{\text{Whole}} \times 100\% = \frac{35}{140} \times 100\% = 25\%.$$

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

Learning Objectives:

- ▶ Study of Logic and examine informal logical fallacies:
 - ▶ Logical arguments
 - ▶ Fallacies of relevance
 - ▶ Fallacies of presumption
 - ▶ Inductive reasoning and pattern recognition

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

- ▶ **Logic** is the study of methods and principles used to distinguish good (correct) from bad (incorrect) reasoning.
- ▶ A logical argument consists of **premises (hypotheses)** and a **conclusion**. The premises are assumptions that we accept as a starting point. The argument is **valid** if the premises justify the conclusion.
- ▶ **Example:** Identify the premises and conclusion of the following argument. Is this argument valid?

All wizards have white beards. Gandalf is wizard.

- ▶ **Solution:** *Therefore, Gandalf has a white beard.*
 - ▶ The premises are: (1) All wizards have white beards and (2) Gandalf is a wizard.
 - ▶ The conclusion is: Gandalf has a white beard. This is a valid argument.
-

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

- ▶ The **fallacy** refers to an argument that may on the surface seem to be correct but is in fact incorrect.
- ▶ An **informal fallacy** is a fallacy that arises from the content of an argument, not its form or structure. The argument is incorrect because of *what* is said, not *how* it is said.
- ▶ A **formal fallacy** arises in the form or structure of an argument. The fallacy is independent of the content of the argument.

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

▶ **Fallacies of relevance:**

the premises are logically irrelevant to, and hence incapable of establishing the truth of, their conclusions.

A. **Appeal to ignorance:**

1. A certain statement is unproven.
2. Therefore, the statement must be false.

▶ **Example:** *For over 75 years people have tried and failed to show that aliens have not visited Earth. So we must finally accept the fact that at least some of the UFO reports are based on actual alien visits.*

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

B. Dismissal based on personal attack:

1. A person presents an argument or point of view.
2. The character of that person is brought into question.
3. Based on the character attack, it is concluded that the argument or point of view is incorrect.

- ▶ **Example:** *My political opponent is against government-funded health care, and she also has a reputation for being heartless. She refused to seek medical treatment for her own father when he was ill. So government-funded health care is a good idea.*

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

C. False authority:

1. A person makes a claim based on his or her authority.
2. The claim is outside the scope of that person's authority.
3. The truth of the argument is concluded based on the authority of the claimant.

▶ **Example:** *Over the past few years, I have starred in a number of the most popular movies in America, so I can assure you that Johnson and Johnson's new anti-nausea drug is medically safe and effective.*

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

D. **Straw man:**

1. A position or point of view is presented.
2. The case for dismissing a distorted or *different* position or point of view (the straw man) is offered.
3. The original position is dismissed on the basis of the reputation of the straw-man position.

▶ **Example:** *A group of my fellow senators is proposing a cut in military expenditures. I cannot support such a cut because leaving our country defenseless in these troubled times is just not acceptable to me.*

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

E. Appeal to common practice:

1. The claim is offered that a position is popular.
2. The validity of the claim is based on its popularity.

▶ **Example:** *It is OK to cheat on your income taxes because everybody does it.*

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

▶ **Example:** Identify the fallacies of relevance.

1. *I have scoured the library for information on witchcraft. I cannot find a single source that proves that anyone accused of witchcraft was actually capable of anything magical. This shows that there is no such thing as a real witch.*
2. *The popular actress Jane Fonda condemned America's involvement in the Vietnam war. So we know that conflict was a civil war in which America should never have been involved.*

▶ **Solution:**

1. We rely on a lack of proof to draw the conclusion. This is appeal to ignorance.
2. This argument involves false authority. Jane Fonda was a popular actress does not lend authority to her opinion about the war.

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

▶ **Fallacies of presumption:**

False or misleading assumptions are either tacitly or explicitly assumed, and these assumptions are the basis of the conclusion.

A. False dilemma:

1. An incomplete or inaccurate list of consequences of not accepting an argument is presented.
2. A conclusion is drawn based on the best (or least bad) of these consequences.

▶ **Example:** *You'd better buy this car or your wife will have to walk to work and your kids will have to walk to school. I know you don't want to inconvenience your family, so let's start the paperwork on the automobile purchase.*

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

B. False cause:

1. Two events occur together, or one follows the other.
2. The fact that the events are related is used to conclude that one causes the other.

▶ **Example:** *Studies have shown that many people on a high-carbohydrate diet lose weight. Therefore, a high-carbohydrate diet leads to weight loss.*

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

c. Circular reasoning or begging the question:

1. A position or argument is offered.
2. The position or argument is concluded to be true based on a restatement of the position.

▶ **Example:** *Establishing government-run health insurance would be a mistake because it is just flat wrong to do it.*

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

D. **Hasty generalization:**

1. A statement is true in several cases that may be atypical.
2. The conclusion that it is generally true or always true is drawn based on the few examples.

▶ **Example:** *I know the quarterback, the tight end, and the center on our football team, and all three are excellent students. The athletes at our university do not shrink their scholarly duties.*

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

Some Common Informal Fallacies

- ▶ **Appeal to ignorance:** A statement is either accepted or rejected because of a lack of proof.
- ▶ **Dismissal based on personal attack:** An argument is dismissed based on an attack on the proponent rather than on its merits.
- ▶ **False authority:** The validity of a claim is accepted based on an authority whose expertise is irrelevant.
- ▶ **Straw man:** A position is dismissed based on the rejection of a distorted or different position.
- ▶ **Appeal to common practice:** An argument for a practice is based on the popularity of that practice.

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

Some Common Informal Fallacies

- ▶ **False dilemma:** A conclusion is based on an inaccurate or incomplete list of alternatives.
- ▶ **False cause:** A causal relationship is concluded based on the fact that two events occur together or that one follows the other.
- ▶ **Circular reasoning:** This fallacy simply draws a conclusion that is really a restatement of the premise.
- ▶ **Hasty generalization:** This fallacy occurs when a conclusion is drawn based on a few examples that may be atypical.

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

▶ **Example:** Classify the following fallacies:

1. *He says we should vote in favor of lowering the sales tax, but he has a criminal record. So I think decreasing the sales tax is a bad idea.*
2. *My dad is a professor of physics, and he says Dobermans make better watchdogs than collies.*

▶ **Solution:**

1. We dismiss a position based on a personal attack.
2. Knowledge of physics does not offer qualification for judging dogs. This is a use of false authority.

Chapter 1 Critical Thinking

1.2 Logic and informal fallacies: Does that argument hold water?

- ▶ **Example:** A certain organism reproduces by cell division. The following table shows the number of cells observed to be present over the first few hours:

Hours	0	1	2	3	4	5	6
Number of cells	1	2	4	8	16	32	64

Describe the pattern shown by the table and suggest a general rule for finding the number of cells in terms of the number of hours elapsed.

- ▶ **Solution:** The number of cells present by raising 2 to the power of the number of hours elapsed.

Hours	0	1	2	3	4	5
Number of cells	$1 = 2^0$	$2 = 2^1$	$4 = 2^2$	$8 = 2^3$	$16 = 2^4$	$32 = 2^5$

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

Learning Objectives:

- ▶ Understand the formal logic used by computers
 - ▶ Operation on statements: Negation, conjunction, disjunction, and implication
 - ▶ Truth tables for complex statements
 - ▶ More on the conditional statements: Converse, inverse, contrapositive
 - ▶ Logic and computers

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ The **negation** of a statement is the assertion that means the opposite of the original statement.

P = He likes dogs.

Not p = He does not like dogs.

By its definition, the negation of a true statement is false, and the negation of a false statement is true.

p	Not p
T	F
F	T

The negation of p is false when p is true.

The negation of p is true when p is false.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ The **conjunction** of two statements is the assertion that *both* are true. We will use the symbol **AND** to denote conjunction.

- ▶ **Example:** Assume that

p = Baseball is 90% mental

q = The other half of baseball is physical

The conjunction is p AND q :

*Baseball is 90% mental **and** the other half is physical.*

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ The truth table for the conjunction (p AND q)

p	q	p AND q
T	T	T
T	F	F
F	T	F
F	F	F

p AND q is true when P is true and q is true.

p AND q is false when P is true and q is false.

p AND q is false when P is false and q is true.

p AND q is false when P is false and q is false.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ Four different cases in the conjunction truth table:
 1. **True conjunction:** *A fever may accompany a cold **and** a headache may accompany a cold.*
 2. **False conjunction:** *You are required to pay income tax **and** pigs often fly.*
 3. **False conjunction:** *Aspirin cures cancer **and** water is wet.*
 4. **False conjunction:** *The president's policies are always best for America **and** Congress never passes an unwise bill.*

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ **Example:** In the past, many health insurance policies did not cover preexisting conditions. They did not cover illness that existed prior to the purchase of the policy. A salesman for such a policy stated:

If you buy this policy, it will cover cases of flu in your family next winter, and it will cover treatment for your wife's chronic arthritis.

Was the salesman telling the truth?

- ▶ **Solution:** The policy did not cover the preexisting arthritic condition, so that part of the conjunction was not true. Conjunctions are true only when both parts are true. The salesman did not speak truthfully.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ The **disjunction** of two statements is the assertion that either one or the other is true (or possibly both). Compare this with the conjunction, where both statements *must* be true. We will use the symbol **OR** to denote disjunction.
- ▶ **Example:** Assume that
 - p = This medication may cause dizziness.
 - q = This medication may cause fatigue.

The disjunction is p OR q :

This medication may cause dizziness or fatigue.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ In mathematics and logic, the word **or** is always used in the inclusive: one or the other, or possibly both.

p	q	p OR q
-----	-----	------------

T	T	T
---	---	----------

T	F	T
---	---	----------

F	T	T
---	---	----------

F	F	F
---	---	----------

p OR q is true when p is true and q is true.

p OR q is true when p is true and q is false.

p OR q is true when p is false and q is true.

p OR q is false when p is false and q is false.



Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ Four different cases in the disjunction truth table:
 1. **True disjunction:** *A fever may accompany a cold **or** a headache may accompany a cold.*
 2. **True disjunction:** *You are required to pay income tax **or** pigs often fly.*
 3. **True disjunction:** *Aspirin cures cancer **or** water is wet.*
 4. **False disjunction:** *The president's policies are always best for America **or** Congress never passes an unwise bill.*

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ **Example:** In my favorite restaurant, the waiter asks if I want butter or sour cream on my baked potato. Is this the inclusive or exclusive **or**?

- ▶ **Solution:** His statement surely means you can have butter, sour cream, or both. Therefore, it is the inclusive **or**.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ **Implication:** A statement of the form “If p then q ” is called an *implication* or a *conditional* statement. We use the same terminology as in the preceding section: p is called the *premise* and q is called the *conclusion*. We denote: $p \rightarrow q$.
- ▶ **Example:** Assume that
 - P = Your average is 90% or more.
 - q = You get an A for the course.

The conditional is $p \rightarrow q$:

If your average is 90% or above, then you get an A for the course.

Premise: Your average is 90% or above

Conclusion: You get an A for the course.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ The truth table for the conditional statement:

p	q	$p \rightarrow q$	
T	T	T	$p \rightarrow q$ is true when p is true and q is true.
T	F	F	$p \rightarrow q$ is false when p is true and q is false.
F	T	T	$p \rightarrow q$ is true when p is false and q is true.
F	F	T	$p \rightarrow q$ is true when p is false and q is false.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ Four different cases in the conditional truth table:
 1. **True conditional:** *If Earth is spherical, then you go from Spain to Japan by traveling west.*
 2. **False conditional:** *If the Moon orbits Earth, then the Moon is a planet.*
 3. **True conditional:** *If Earth is the planet nearest to the Sun, then Earth orbits the Sun once each year.*
 4. **True conditional:** *If Earth is the planet farthest from the Sun, then the Moon is made of green cheese.*

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ **Example:** The new president promises,
*If Congress passes my economic package,
then the recession will end within two years.*

Under which of the following scenarios would we think the president's promise was kept?

- ▶ **Scenario 1:** Congress passes the economic passage, but two years later the recession persists.
- ▶ **Scenario 2:** Congress does not pass the economic package, but two years later the recession persists.
- ▶ **Solution:** In Scenario 1, the premise is true but the conclusion is false. The president's promise was not kept. In Scenario 2, the premise of the conditional is false. The promise should be considered to be kept.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ **Example:** Let m represent the statement *Bill is a math major*, and let c represent the statement *Bill is a chemistry major*.
 1. Represent these combinations of statements in symbolic form.
 2. Determine which are true under the assumption that Bill is a math major but not a chemistry major.
 - a. Bill is a math major or a chemistry major.
 - b. Bill is not a math major and is a chemistry major.
 - c. If Bill is not a math major, then he's a chemistry major.
- ▶ **Solution:**
 - a. m OR c : the disjunction is true.
 - b. (NOT m) AND c : the conjunction is false.
 - c. (NOT m) \rightarrow c : the conditional is true.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ **Truth table for complex statements:** We can use the basic truth table presented above to analyze more complex statements.
- ▶ **Example:** Consider a statement
*If you do not remember the past,
then you are condemned to repeat it.*

Let

p = You remember the past.

q = You are condemned to repeat the past.

Make a truth table for $(\text{NOT } p) \rightarrow q$.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- **Solution:** The truth table for $(\text{NOT } p) \rightarrow q$

p	q	$\text{NOT } p$	$(\text{NOT } p) \rightarrow q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

For example, $(\text{NOT } p) \rightarrow q$ is true when both p is true and q is true. This fact is relevant to those people who remember the past and are condemned to repeat it.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ **Example:** In 1994 California voters approved Proposition 187, which included the statement: *A person shall not receive any public social services until he or she has been verified as a United States citizen or as a lawfully admitted alien.*

The law was later judged unconstitutional by a federal court.

Let

C = *Citizenship has been verified.*

A = *Lawfully admitted alien status has been verified.*

Then under Proposition 187, the condition that would deny services is NOT (C or A). Use a truth to analyze this statement.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

► **Solution:**

C	A	C or A	NOT (C or A)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ **More on the conditional: Converse, inverse, contrapositive**—The conditional is the combination of statements that is most often misinterpreted, so it deserves further attention.

Assume that:

p = You support my bill.

q = You are a patriotic American.

$p \rightarrow q$: **If** *you support my bill*, **then** *you are a patriotic American*.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

▶ **The conditional:** $p \rightarrow q$.

If *you support my bill*, **then** *you are a patriotic American*.

1. **The converse:** $q \rightarrow p$.

If *you are a patriotic American*, **then** *you support my bill*.

2. **The inverse:** $(\text{NOT } p) \rightarrow (\text{NOT } q)$.

If *you do not support my bill*, **then** *you are not a patriotic American*.

3. **The contrapositive:** $(\text{NOT } q) \rightarrow (\text{NOT } p)$.

If *you are not a patriotic American*, **then** *you do not support my bill*.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

		NOT	NOT	Conditional	Converse	Inverse	Contrapositive
p	q	p	q	$p \rightarrow q$	$q \rightarrow p$	$(\text{NOT } p) \rightarrow (\text{NOT } q)$	$(\text{NOT } q) \rightarrow (\text{NOT } p)$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

- ▶ Two statements are **logically equivalent** if they have the same truth table.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

Conditional Statements

Consider the conditional statement **If** *statement A*, **then** *statement B*.

- The contrapositive statement is

If *statement B is false*, **then** *statement A is false*.

A conditional is logically equivalent to its contrapositive.

- The converse statement is

If *statement B*, **then** *statement A*.

- The inverse statement is

If *statement A is false*, **then** *statement B is false*.

The inverse is logically equivalent to the converse.

- Replacing a conditional statement with its inverse or converse is **not** logically valid.

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

▶ **Example:** Formulate the converse, inverse, and contrapositive of each of the following conditional statements:

1. *If you vote for me, your taxes will be cut.*
2. *All Democrats are liberals.*

▶ **Solution:**

1. The converse: *If your taxes are cut, you voted for me.*

The inverse: *If you do not vote for me, your taxes will not be cut.*

The contrapositive: *If your taxes are not cut, you did not vote for me.*

2. The converse: *If you are a liberal, you are a Democrat.*

The inverse: *If you are not a Democrat, you are not a Liberal.*

The contrapositive: *If you are not a liberal, you are not a Democrat.*

Chapter 1 Critical Thinking

1.3 For logic and truth tables: Do computers think?

- ▶ **Example:** The logic gate AND takes two inputs, each of them 0 or 1. It corresponds to the logical operation of conjunction. If the inputs to the AND gate are 1 and 1, what is the output?
- ▶ **Solution:** This situation corresponds to the conjunction p AND q when both p and q are true. The output of the logic gate is 1.

Chapter 1 Critical Thinking: **Chapter Summary**

- ▶ **Public policy and Simpson's paradox: Is "average" always average?**
 - ▶ Understand that Simpson's paradox is a striking example of the need for critical thinking skills.
 - ▶ Overall average may lead to *invalid* conclusion.
- ▶ **Logic and informal fallacies: Does that argument hold water?**
 - ▶ Logical argument involves: Premises, Conclusion
 - ▶ Informal fallacies: *fallacies of relevance*,
fallacies of presumption
 - ▶ Deductive arguments and Inductive arguments

Chapter 1 Critical Thinking: **Chapter Summary**

- ▶ Formal logic and truth tables: Do computers think?
 - ▶ Formal logic: The truth table
 - ▶ Operations on statements: Negation, conjunction, disjunction, conditional or implication.